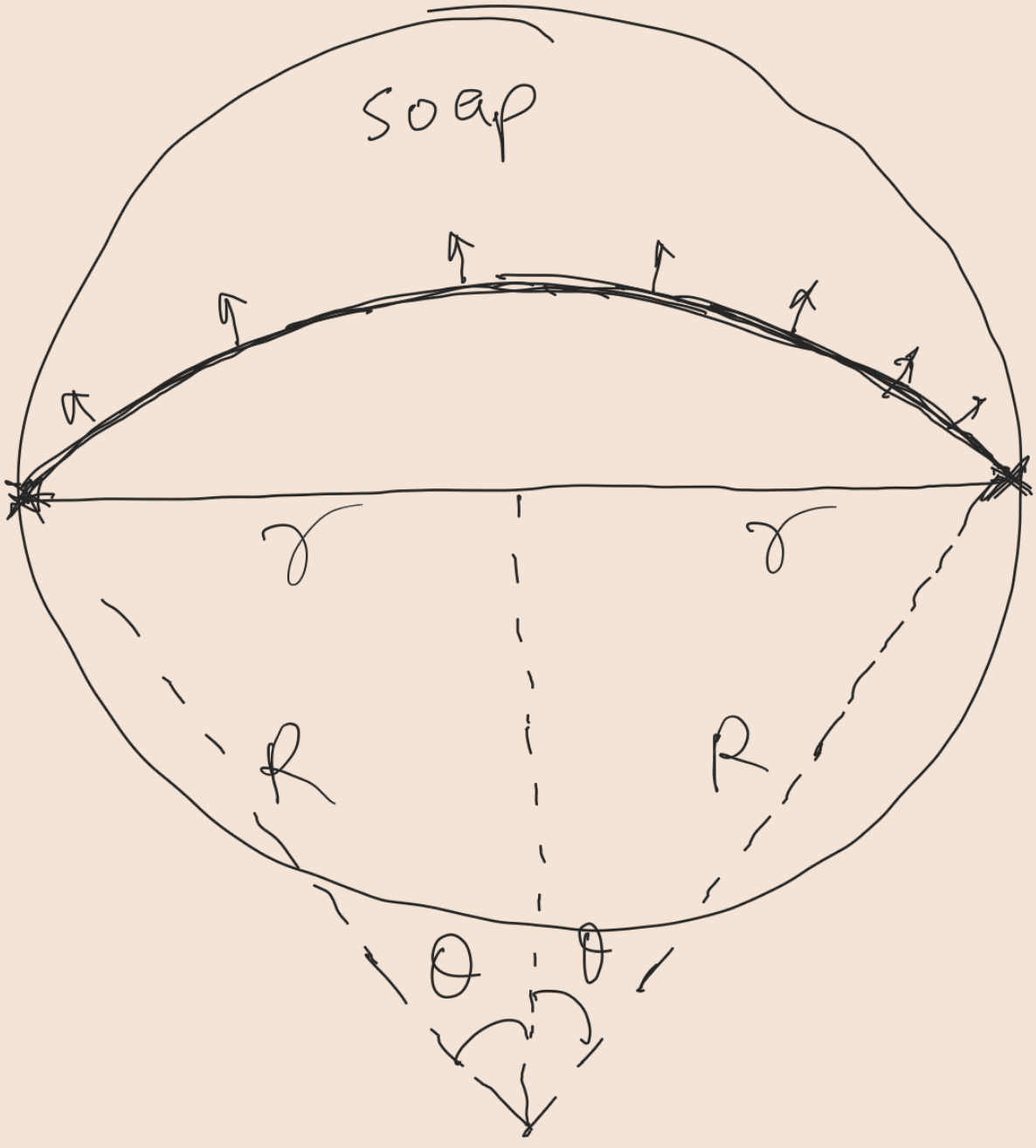
Situation after puncture



Thread will assume the shape of circular arc due to uniform surface tension stretching force at all its elements.

$$r = R\sin\theta \qquad \theta = \sin^{-1}\left(\frac{r}{R}\right)$$

$$\theta = \frac{r}{R} + \frac{1}{6}\frac{r^3}{R^3}.$$

$$2R\theta = 2r + \frac{1}{3}\frac{r^3}{R^2}.$$

$$2R\theta - 2r = \delta\,(\text{elongation})$$

$$= \frac{1}{3}\frac{r^3}{R^2}.$$

Let's draw FBD of small element of thread.



$$2F\left(\frac{d\phi}{2}\right) = 2\pi R\left(\frac{d\phi}{2\pi}\right)$$

$$F = 2\pi R. \quad \text{Also,}$$

$$\frac{F}{S} = Y \frac{\delta}{2r} \; ; \quad \frac{2\pi R}{S} = Y \frac{\delta}{2r}$$

$$R = Y \frac{\delta S}{4r\sigma} \; . \quad \text{Back}$$

Substituting in original equation, $\delta = \frac{1}{3} \frac{r^3}{\left(\frac{Y^2 \delta^2 S^2}{16 r^2 \sigma^2}\right)}$

$$\delta = \left(\frac{16 \sigma^2 r^5}{3 Y^2 S^2}\right)^{\frac{1}{3}}$$